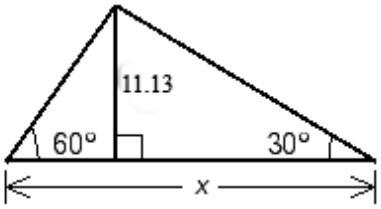
Math 5 – Trigonometry – Chapter 5 Test Solutions – fall '07

1. The angle of elevation to the top of the Diesel Mechanics building from a point 82 feet from the base is 0.5 radians. Approximate the height the Diesel Mechanics building to the nearest foot.

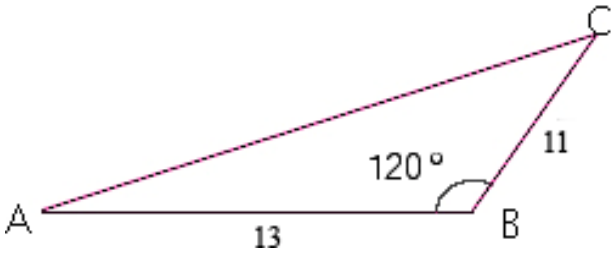
   SOLN: $\tan(0.5) = \dfrac{h}{82} \Rightarrow h \approx 0.5463(82) \approx 45$ ft.

2. Find x correct to four significant digits.



SOLN: $x = \dfrac{11.13}{\sqrt{3}} + 11.13\sqrt{3} \approx 25.70$

3. Find the area of the triangle at right:



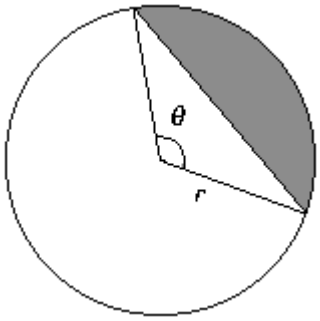
   SOLN: A = 11(13)sin(120°)/2 =

   $= \dfrac{143\sqrt{3}}{4} \approx 61.92$

4. If $\theta = \dfrac{2\pi}{3}$ find the following.

   a. $\tan^2(\theta) = \tan^2\left(\dfrac{2\pi}{3}\right) = \left(\sqrt{3}\right)^2 = 3$

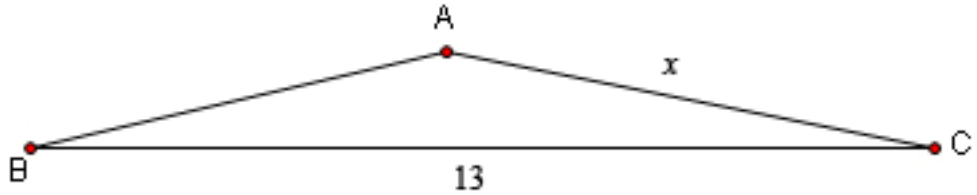
   b. $\tan\left(\theta^2\right) = \tan\left(\dfrac{4\pi^2}{9}\right) \approx \tan 4.3865 \approx 2.959$



5. Find the area of the shaded region in the figure if $r = 13$ and $\theta = 110°$.
   SOLN:
   Area of sector – area of triangle =

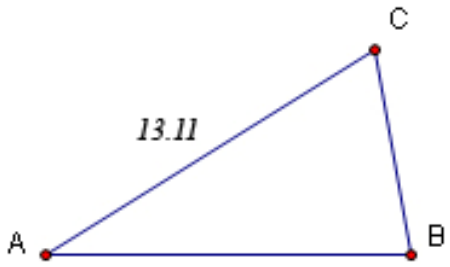
$$\frac{r^2\theta}{2} - \frac{1}{2}r^2\sin\theta = \frac{13^2 11\pi}{2(18)} - \frac{13^2}{2}\sin\left(\frac{11\pi}{18}\right) = \frac{1859\pi}{36} - \frac{169}{2}\sin\left(\frac{11\pi}{18}\right) \approx 162.2 - 79.4 \approx 82.8$$

6. Find the value of x in the diagram below. Assume that $\angle ABC = 13^\circ$ and $\angle ACB = 11^\circ$



SOLN: $\dfrac{x}{\sin 13^\circ} = \dfrac{13}{\sin 156^\circ} \Leftrightarrow x = \dfrac{13\sin 13^\circ}{\sin 156^\circ} \approx \dfrac{13(0.22495)}{(0.40674)} \approx 7.190$

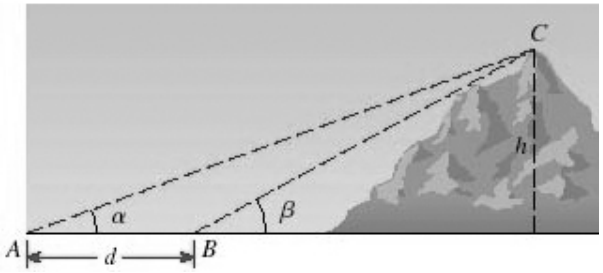
7. Sketch the triangle with $\angle A = 32^\circ$, $\angle C = 68^\circ$ and $b = 13.11$, then solve the triangle.



SOLN: $a = \dfrac{13.11\sin 32^\circ}{\sin 80^\circ} \approx 7.054$, $\angle b = 80^\circ$ and

$c = \dfrac{13.11\sin 68^\circ}{\sin 80^\circ} \approx 12.34$

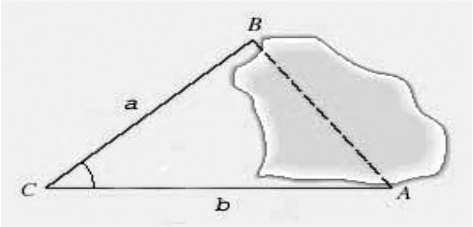
8. To calculate the height of a mountain, angles $\alpha = 11°$, $\beta = 13°$ and $d = 311$ ft are measured. Use the formula $h = d\dfrac{\sin\alpha\sin\beta}{\sin(\beta-\alpha)}$ to calculate the height.



SOLN: $h = d\dfrac{\sin\alpha\sin\beta}{\sin(\beta-\alpha)} = \dfrac{311\sin 11^\circ\sin 13^\circ}{\sin 2^\circ} \approx \dfrac{311(0.19081)(0.22495)}{0.03490} \approx 382.5$ ft

(Not much of a mountain.)

9. To find the distance across a lake, a surveyor has taken the measurements $a = 11$ mi., $b = 13$ mi. and $\angle C = 40^\circ$. Find the distance across the lake using this information. Round to 2 significant digits.



SOLN: By the law of cosines, the square of the distance is

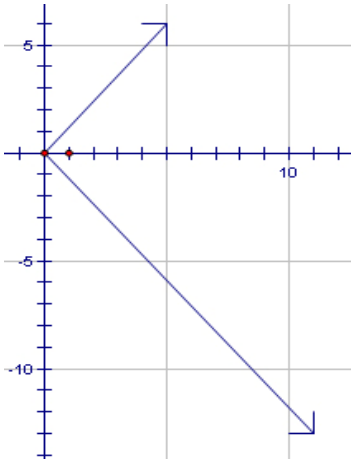
$11^2 + 13^2 - 2(11)(13)\cos(40^\circ) \approx 290 - 286(0.76604) \approx 70.91$ So $AB \approx 8.421$

10. A toy bicycle with one wheel of diameter 11cm and a bigger wheel with diameter 13cm is rolling along so that the big wheel is rolling at 10 rotations per minute. What is the angular speed of the little wheel?

SOLN: First find the linear speed of the bike: $v = \omega r = \dfrac{10\text{rotations}}{\text{min}} \times \dfrac{2\pi}{\text{rotation}} \times 6.5\text{cm} = 130\pi\dfrac{\text{cm}}{\text{min}}$

Then find the angular velocity of the little wheel: $\omega = \dfrac{v}{r} = \dfrac{130\pi\text{cm/min}}{5.5\text{cm}} = \dfrac{260\pi}{11}\text{rad/min} \approx 74\text{rad/min}$

11. Suppose we have vectors $\vec{u} = 5\hat{i} + 6\hat{j}$ and $\vec{v} = 11\hat{i} - 13\hat{j}$



a. Draw and label these vectors together in the *x*-*y* plane, assuming each has its initial point at (0,0). (SOLN: See diagram at right.)

b. Find the angle between these two vectors.
SOLN: First find the lengths of the vectors:
$\left|\langle 11,-13\rangle\right| = \sqrt{11^2+13^2} = \sqrt{290}$ and $\left|\langle 5,6\rangle\right| = \sqrt{61}$ so that

$\theta = \cos^{-1}\left(\dfrac{\langle 11,-13\rangle\cdot\langle 5,6\rangle}{\sqrt{61}\sqrt{290}}\right) \approx \cos^{-1}\left(\dfrac{-23}{133}\right) \approx 100^\circ$

c. Find the length of $\vec{u}$ and the length of $\vec{v}$. (SOLN: See above.)

d. $\left|\vec{u}+\vec{v}\right| = \left|\langle 16,-7\rangle\right| = \sqrt{256+49} = \sqrt{305}$ and $\left|\vec{u}-\vec{v}\right| = \left|\langle \vec{u}-\vec{v}\rangle\right| = \sqrt{397}$

12. Suppose $\vec{v} = 11\hat{i} - 13\hat{j}$. Find a value of b so that the vector $\vec{u} = 10\hat{i} + b\hat{j}$ is orthogonal to *v*.

SOLN: $\vec{v} \cdot \vec{u} = \langle 11, -13 \rangle \cdot \langle 10, b \rangle = 110 - 13b = 0 \Leftrightarrow b = \frac{110}{13}$